37. Vectors in two dimensions

 describe a translation by using a vector represented by

$$\begin{pmatrix} x \\ y \end{pmatrix}$$
, \overrightarrow{AB} or **a**

- · add and subtract vectors
- · multiply a vector by a scalar
- calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$
- represent vectors by directed line segments
- use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors
- · use position vectors

Vectors will be printed as \overline{AB} or \mathbf{a} and their magnitudes denoted by modulus signs, e.g. $|\overline{AB}|$ or $|\mathbf{a}|$.

In their answers to questions candidates are expected to indicate **a** in some definite way, e.g. by an arrow \overrightarrow{AB} or by underlining as follows **a**.

Vectors

M/J19/12/Q25

1 (a) $P = \begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix} \qquad Q = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$

Evaluate PQ.



(b) $\mathbf{M} = \begin{pmatrix} 3 & -1 \\ 2 & k \end{pmatrix}$

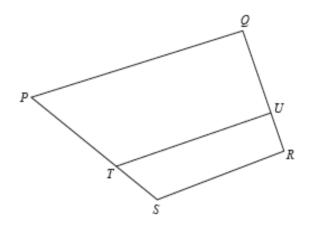
The determinant of matrix M is -4.

(i) Find the value of k.

(ii) Find M^{-1} .

M/J19/21/Q9

2 (a)



NOT TO SCALE

In the diagram, $\overrightarrow{PQ}=4\mathbf{p}$, $\overrightarrow{QR}=3\mathbf{q}$ and $\overrightarrow{PT}=\mathbf{p}+2\mathbf{q}$. $\overrightarrow{QU}=\frac{2}{3}\overrightarrow{QR}$ and $\overrightarrow{PT}=\frac{2}{3}\overrightarrow{PS}$.

- (i) Express, as simply as possible, in terms of p and/or q,
 - (a) \$\overline{PS}\$,

$$\overrightarrow{PS} = \dots$$
 [1]

(b) *SR* .

$$\overrightarrow{SR} = \dots$$
 [2]

State the name of the special quadrilateral PQRS.
 Using vectors, give a reason for your answer.

(iii) Find, in its simplest form, the ratio $|\overrightarrow{PQ}|$: $|\overrightarrow{SR}|$.

(b)
$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$
 $\overrightarrow{BC} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ $\overrightarrow{CD} = \begin{pmatrix} -7 \\ -3 \end{pmatrix}$

(i) Find \overrightarrow{AD} .

$$\overrightarrow{AD} = \left(\right)$$
 [1]

(ii) Find \overrightarrow{BC} .

.....[2]

(iii) Given that E is the midpoint of BC, find \(\overline{AE}\).

$$\overrightarrow{AE} = \begin{pmatrix} \\ \end{pmatrix}$$
 [2]

M/J19/22/Q10

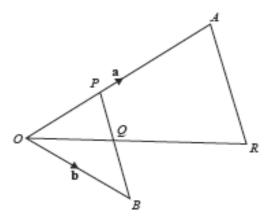
- 3 (a) $f = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $g = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$
 - (i) Find g − 2f.

() [1]

(ii) Petra writes |f| > |g|.
Show that Petra is wrong.

[3]

(b)



NOT TO SCALE

 $O, A \text{ and } B \text{ are points with } \overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}.$

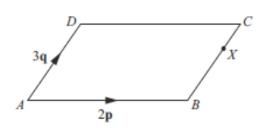
P is the point on OA such that $OP = \frac{1}{3}OA$.

- O, Q and R lie on a straight line and Q is the midpoint of PB.
- Find PB in terms of a and b.

(ii)	Find \overrightarrow{OQ} in terms of a and b. Give your answer in its simplest form.		
		$\overrightarrow{OQ} = \dots$	[2]
iii)	QR = 2OQ.		
	Show that AR is parallel to PB.		

SM18/12/Q7

4



ABCD is a parallelogram.

X is the point on BC such that BX : XC = 2 : 1.

$$\overrightarrow{AB} = 2\mathbf{p}$$
 and $\overrightarrow{AD} = 3\mathbf{q}$.

Find, in terms of p and q,

(a)
$$\overrightarrow{AC}$$
,

Answer
$$\overrightarrow{AC}$$
 =[1]

(b)
$$\overrightarrow{AX}$$
,

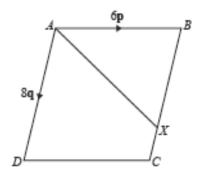
Answer
$$\overrightarrow{AX}$$
 =[1]

(c)
$$\overrightarrow{XD}$$
.

Answer
$$\overrightarrow{XD}$$
 =[1]

O/N18/11/Q23

5



In the diagram, ABCD is a parallelogram. X is the point on BC such that BX:XC=3:1. $\overrightarrow{AB}=$ 6p and $\overrightarrow{AD}=$ 8q.

(a) Express BX in terms of p and/or q.

Answer		[1]	l
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(b) Express \overrightarrow{AX} in terms of p and/or q.

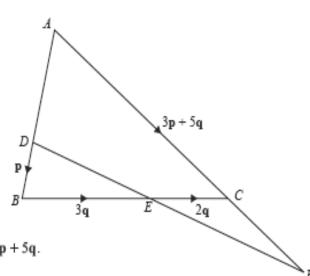
- (c) Y is the point such that $\overrightarrow{CY} = 3p + q$.
 - (i) Express \overrightarrow{AY} in terms of p and/or q.

(ii) Find the ratio AX: AY.

W18/12/Q25

6 In the diagram, ADB and ACF are straight lines.

BC intersects DF at E.



AC: CF = 2:1.

$$\overrightarrow{DB} = \mathbf{p}$$
, $\overrightarrow{BE} = 3\mathbf{q}$, $\overrightarrow{EC} = 2\mathbf{q}$ and $\overrightarrow{AC} = 3\mathbf{p} + 5\mathbf{q}$.

(a) Express \overrightarrow{AB} in terms of p.

Answer
$$\overrightarrow{AB} = \dots$$
 [1]

(b) Express \overrightarrow{CF} in terms of p and/or q.

Answer
$$\overrightarrow{CF} = \dots$$
 [1]

(c) Express \overrightarrow{EF} in terms of \mathbf{p} and/or \mathbf{q} .

Answer
$$\overrightarrow{EF} = \dots [1]$$

(d) $\overrightarrow{EF} = k\overrightarrow{DE}$.

Find k.

O/N18/21/Q7

- 7 The position vector, \overrightarrow{OA} , of point A is $\begin{pmatrix} -4 \\ 7 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$.
 - (a) Find the position vector, \overrightarrow{OB} , of point B.

Answer
$$\overrightarrow{OB} = \begin{pmatrix} \\ \\ \end{pmatrix}$$
 [1]

(b) Find $|\overrightarrow{AB}|$.

(c) Given that $\overrightarrow{AB} = 3\overrightarrow{CB}$, find the coordinates of point C.

Answer (.....) [2]

(d)	Line L is parallel to \overrightarrow{AB} and passes through the point (-2, 5).				
	(i)	Find the equation of line L .			
		Answer[3]			
	(ii)	Line M is perpendicular to line L and passes through the origin.			
		Find the equation of line M .			
		Answer[1]			

M/J18/11/Q21

$$\mathbf{p} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad \mathbf{q} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

(a) Write $3\mathbf{p} - \mathbf{q}$ as a column vector.

Find the value of n.

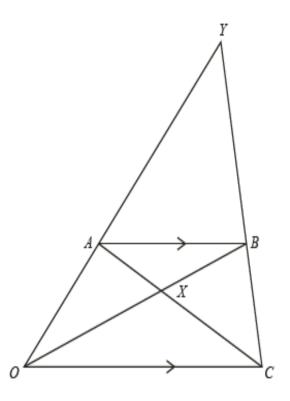
Answer () [1]

(b) R is the point (11, -2) and O is the point (0, 0). The vector \overrightarrow{OR} can be written in the form $\mathbf{p} + n\mathbf{q}$, where n is an integer.

Answer $n = \dots [2]$

M/J18/22/Q8

9



OYC is a triangle.

A is a point on OY and B is a point on CY.

AB is parallel to OC.

AC and OB intersect at X.

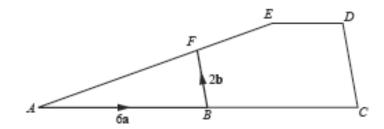
(a)	Prove that triangle ABX is similar to triangle COX.
	Give a reason for each statement you make.

 		Г3

(b)	\overrightarrow{OA}	= 3a and \overrightarrow{OC} = 6c and $CB : BY = 1 : 2$.		
	Find	l, as simply as possible, in terms of a and/or c		
	(i)	\overrightarrow{AB} ,		
	(ii)	\overrightarrow{CY} .	Answer	$\overrightarrow{AB} = \dots [1]$
			Answer	$\overrightarrow{CY} = \dots [2]$
(c)	Find	l, in its simplest form, the ratio		
	(i)	OX: XB,		
			4	
	(ii)	area of triangle COX : area of triangle ABX ,	Answer	[2]
(iii)	area of triangle AYB : area of trapezium $OABC$.	Answer	[1]
			Answer	· [11

O/N17/11/Q24

10



In the diagram, ABC and AFE are straight lines.

$$\overrightarrow{AB} = 6a$$
 and $\overrightarrow{BF} = 2b$.

(a) Express \overrightarrow{AF} in terms of a and b.

- (b) $\overrightarrow{AE} = 9a + kb$.
 - Find k.

Answer
$$k = \dots [1]$$

(ii) ED is parallel to BC, CD is parallel to BF and BC = AB.

Find, in terms of a and/or b,

(a) \$\overline{CD}\$,

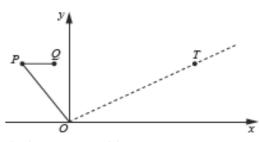
Answer[1]

(b) DE.

Answer[1]

O/N17/12/Q27

11



In the diagram, $\overrightarrow{OP} = \begin{pmatrix} -3\\4 \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} 2\\0 \end{pmatrix}$.

(a) Find $|\overrightarrow{OP}| + |\overrightarrow{PQ}|$.

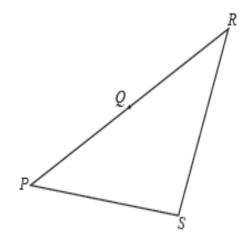
Answer[3]

- (b) T is the point where $\overrightarrow{PT} = k\overrightarrow{PQ}$.
 - (i) Express \overrightarrow{OT} as a column vector in terms of k.

(ii) M is the point such that O, T and M lie on a straight line and $\overrightarrow{OM} = \begin{pmatrix} 24 \\ 16 \end{pmatrix}$. Find the value of k.

O/N17/21/Q10(b)

12 The diagram shows triangle PRS.



Q is the midpoint of PR.

$$\overrightarrow{PQ} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$
 and $\overrightarrow{PS} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$.

(i) Find \overrightarrow{SR} .

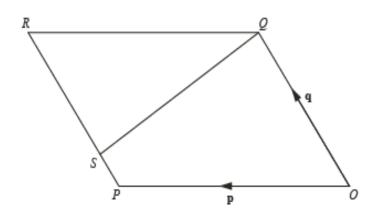
Answer [2]

(ii) T is the point on SR such that ST: TR = 1:3.
Find \(\overline{PT} \).

Answer [2]

M/J17/11/Q23

13



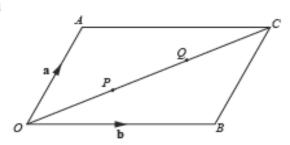
OPRQ is a parallelogram and S is a point on PR such that PS: SR = 1:3.

$$\overrightarrow{OP} = \mathbf{p}$$
 and $\overrightarrow{OQ} = \mathbf{q}$.

- (a) (i) Express \overline{PQ} in terms of p and/or q.
- Answer[1]
- (ii) Express \overline{QS} , as simply as possible, in terms of **p** and/or **q**.
 - Answer[1]
- (b) T is a point on QS extended such that $\overline{QT} = \frac{4}{3} \overline{QS}$.
 - (i) Express \overrightarrow{PT} , as simply as possible, in terms of p and/or q.
 - Answer[2]
 - (ii) What can you conclude about the points O, P and T?
 -[1]

M/J17/12/Q21

14



OACB is a parallelogram.

 $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$.

P and Q are points on OC such that OP = PQ = QC.

- (a) Express, as simply as possible, in terms of a and b,
 - (i) OP,

Answer .	 Π	1

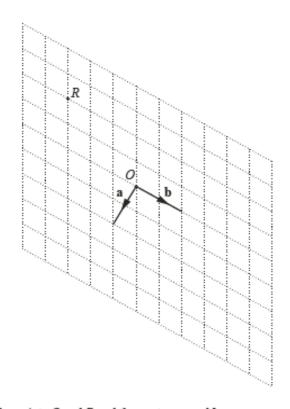
(ii) BP.

A	nswer	 []	ľ]	

(b) Show that triangles OAQ and CBP are congruent.

O/N16/11/Q19

15



The diagram shows the points O and R and the vectors \mathbf{a} and \mathbf{b} .

(a) Given that
$$\overrightarrow{OP} = 2a$$
, mark and label the position of P on the grid. [1]

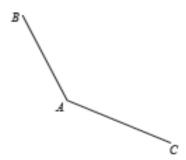
(b) Given that
$$\overline{QQ} = 2\mathbf{b} - \mathbf{a}$$
, mark and label the position of Q on the grid. [1]

(c) Express \overrightarrow{OR} in terms of a and b.

Answer
$$\overline{OR} = \dots$$
 [2]

O/N16/21/Q11(a)

16 (a)



In the diagram, $\overrightarrow{AB} = \begin{pmatrix} -6 \\ 11 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$.

Find | AC |.

Answer[2]

- (ii) D is the point such that $\overline{AD} = \begin{pmatrix} 0 \\ k \end{pmatrix}$, where k > 0. BD is parallel to AC.
 - (a) Show that $\overrightarrow{BD} = \begin{pmatrix} 6 \\ k-11 \end{pmatrix}$.

[1]

(b) Find k.

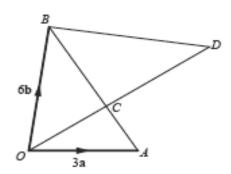
Answer k =[2]

(c) Find the difference between the lengths of AD and AC.

Answer[1]

O/N16/Q10(a)

17 (a)



	ACB	and	OCD	аге	straight	lines
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AC:CB=1:2.

 $\overrightarrow{OA} = 3a$ and $\overrightarrow{OB} = 6b$.

(i) Express \overrightarrow{AB} in terms of a and b.

Answer[1]

(ii) Express \overrightarrow{AC} in terms of a and b.

Answer [11]

(iii) $\overrightarrow{BD} = 5\mathbf{a} - \mathbf{b}$.

Showing your working clearly, find OC: CD.

Answer [4]

MARKING SCHEME

Question	Answer	Marks	Partial Marks
1(a)	$\begin{pmatrix} 4 & 8 \\ -2 & -7 \end{pmatrix}$	2	B1 for two or three correct elements
1(b)(i)	-2	1	
1(b)(ii)	$-\frac{1}{4}\begin{pmatrix} -2 & 1\\ -2 & 3 \end{pmatrix} \text{ oe isw}$	1	$\mathbf{FT} -\frac{1}{4} \begin{pmatrix} their k & 1 \\ -2 & 3 \end{pmatrix}$
	or $\begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{pmatrix}$ oe isw		

	i		ı
2(a)(i)(a)	$\frac{3}{2}$ (p + 2q) oe simplified expression	1	
2(a)(i)(b)	$\frac{5}{2}\mathbf{p} \text{ or } 2\frac{1}{2}\mathbf{p}$ or 2.5 \mathbf{p}	2	M1 for a correct vector route
2(a)(ii)	Trapezium	B1	
	\overrightarrow{PQ} is a multiple of \overrightarrow{SR} or PQ is parallel to SR since \overrightarrow{PQ} =4p and \overrightarrow{SR} =2.5p oe	B1	
2(a)(iii)	8:5	2	FT their \overrightarrow{SR} of form $k\mathbf{p}$ B1 for 4: 2.5 oe
2(b)(i)	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ final answer	1	
2(b)(ii)	6.32 or 6.324 to 6.325	2	M1 for $6^2 + (-2)^2$
2(b)(iii)	$\binom{6}{1}$ final answer	2	B1 for $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$
1			

3(a)(i)	$\begin{pmatrix} -7 \\ 1 \end{pmatrix}$ final answer	1	

Question	Answer	Marks	Partial Marks
3(a)(ii)	$4^2 + (\pm 3)^2$	M1	
	$1^2 + (\pm 5)^2$	M1	
	Correct concluding statement eg	A1	
	$\sqrt{25} < \sqrt{26} \text{ or }$		
	5 > 5.1[0] wrong or		
	f = 5 $ g = 5.099$ so $ f $ is not greater than $ g $		

Question	Answer	Marks	Part marks
4(a)	$2\mathbf{p} + 3\mathbf{q}$	1	
4(b)	$2\mathbf{p} + 2\mathbf{q}$	1	
4(c)	$-2\mathbf{p}+\mathbf{q}$ ft	1	Accept 3q – their (b) ft

5(a)	6 q oe	1	
5(b)	6 p + 6 q isw	1	FT 6p + their (a) isw
5(c)(i)	$9\mathbf{p} + 9\mathbf{q}$ oe	1	
5(c)(ii)	2:3 oe	1	

6(a)	3 p	1	
6(b)	$\frac{1}{2}(3p+5q)$ oe	1	
6(c)	$\frac{1}{2}(3\mathbf{p} + 9\mathbf{q})$ oe	1	FT 2q oe + their (b) isw
6(d)	1.5 oe	2	B1 for $[\overrightarrow{DE} =] \mathbf{p} + 3\mathbf{q}$; or for $k(\mathbf{p} + 3\mathbf{q})$

Question	Answer		Mar	ks	Partial Marks
7(a)	$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$			1	
7(b)	6.71 or 6.708			2	M1 for $6^2 + (-3)^2$ oe
7(c)	(0, 5)			2	FT their (a) ((their 2 – 2), (their 4 + 1))
					B1 for one value in coordinates correct or for $\left[\overline{CB} = \right] \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ soi
7(d)(i)	$y = -\frac{1}{2}x + 4$ oe final answer			3	B2 for $y = -\frac{1}{2}x + c$ oe OR M1 for gradient = $\frac{-3}{6}$ soi M1 for (-2, 5) substituted into
					y = their mx + c
7(d)(ii)	y = 2x oe			1 FT their gradient from (d)(i)	
-	+	+		+	
8(a)	$\begin{pmatrix} 13 \\ 9 \end{pmatrix}$		1		
8(b)	n = -2		2		M1 for $\begin{pmatrix} 3\\4 \end{pmatrix} + n \begin{pmatrix} -4\\3 \end{pmatrix} = \begin{pmatrix} 11\\-2 \end{pmatrix}$ or $3 + (-4n) = 1$ or $4 + 3n = -2$
		1	1	ı	1
9(a)	$\angle BAX = \angle OCX$, alternate [angles] $\angle ABX = \angle COX$, alternate [angles] $\angle AXB = \angle CXO$, [vertically] opposite				for two correct pairs of angles for correct reason for one pair of angles
9(b)(i)	4 c		1		
9(b)(ii)	9a – 6c or 3(3a – 2c)		2 E	31 f	For answer $9\mathbf{a} + k\mathbf{c}$ or $k\mathbf{a} - 6\mathbf{c}$ $(k \neq 0)$

1

3:2

9:4

4:5

9(c)(i)

9(c)(ii)

9(c)(iii)

2 B1 for 3k : 2k, where k is an integer

1 FT their 3^2 : their 2^2

10(a)	6 a + 2 b oe	1				
10(b)(i)	3	1				
10(b)(ii)(a) $3\mathbf{b}$; or FT $k\mathbf{b}$	1				
10(b)(ii)(b) -3 a	1				
11(a)	7	3	3 M1 for $ \overline{OP} = \sqrt{(-3)^2 + (4)^2}$ B1 for $ \overline{PQ} = 2$			$(-3)^2 + (4)^2$
11(b)(i)	$\begin{pmatrix} -3+2k\\4 \end{pmatrix} \text{ oe }$	1				
11(b)(ii	$4\frac{1}{2}$ oe	2	1			\overrightarrow{OM} as a multiple (by 4) of \overrightarrow{OT} 4); or for $\overrightarrow{OT} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$
12(i)	12(i) $ \begin{pmatrix} 4 \\ 8 \end{pmatrix} $ 2 B1 for one component correct or M1 for $2\binom{6}{3} - \binom{8}{-2}$ oe After 0 scored, SC1 for answer $\binom{-4}{-8}$			$2\binom{6}{3} - \binom{8}{-2}$ oe		
12(ii)	$\begin{pmatrix} 9 \\ 0 \end{pmatrix}$		2			component correct $-\frac{3}{4}(their \ \overline{SR}) \text{ or } \frac{1}{4}(their \ \overline{SR}) \text{ so}$
				+		
13(a)(i)	q – p				1	
13(a)(ii)	$\mathbf{p} - \frac{3}{4}\mathbf{q} \text{ or } \frac{4\mathbf{p} - 3\mathbf{q}}{4}$				1	
13(b)(i)	$\overrightarrow{PT} = \frac{1}{3}\mathbf{P}$				2	M1 for $\overrightarrow{PT} = \overrightarrow{PS} + \frac{1}{3} \overrightarrow{QS}$ soi
						or $\overrightarrow{PT} = PQ + QT$ soi
13(b)(ii)	O, P and T are collinear oe				1	e.g. T is on OP produced

14(a)(i)	$\frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \text{ or } \frac{1}{3}(\mathbf{a} + \mathbf{b}) \text{ or } \frac{\mathbf{a} + \mathbf{b}}{3}$ final answer	1	
14(a)(ii)	$\frac{1}{3}\mathbf{a} - \frac{2}{3}\mathbf{b} \text{ or } \frac{1}{3}(\mathbf{a} - 2\mathbf{b}) \text{ or } \frac{\mathbf{a} - 2\mathbf{b}}{3}$ final answer	1	
14(b)	Any two pairs of vectors from $\overrightarrow{OA} = \overrightarrow{BC}$ oe $\overrightarrow{OQ} = \overrightarrow{PC}$ oe $\overrightarrow{QA} = \overrightarrow{BP}$ oe	2	B1 for any one pair of vectors stated
	Alternative method: OA = BC OQ = PC $\angle AOQ = \angle BCP$		B1 for two of these pairs of sides stated or one of these pairs of sides and this pair of angles stated
	+		+

15 (a)	the point P marked correctly	1	
(b)	the point Q marked correctly	1	
(c)	- a − 2 b oe	2	C1 for -a ; or for -2b

16	(a)	(i)	13	2	M1 for $\sqrt{(-5)^2 + 12^2}$
		(ii) (a)	$[\overrightarrow{BD} =]\overrightarrow{BA} + \overrightarrow{AD} = \begin{pmatrix} 6 \\ -11 \end{pmatrix} + \begin{pmatrix} 0 \\ k \end{pmatrix} = \begin{pmatrix} 6 \\ k - 11 \end{pmatrix}$ \mathbf{AG}	1	Or $[\overrightarrow{BD} =]\overrightarrow{AD} - \overrightarrow{AB} = \begin{pmatrix} 0 \\ k \end{pmatrix} - \begin{pmatrix} -6 \\ 11 \end{pmatrix} = \begin{pmatrix} 6 \\ k - 11 \end{pmatrix}$

17 (a) (i)	6 b – 3 a oe isw	1	
(ii)	$2\mathbf{b} - \mathbf{a}$ oe isw	1ft	
(iii)	2:3 cao NB www	4	M1+ M1 for two of $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ $\overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{BD}$ $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$ A1 for $\overrightarrow{OC} = 2\mathbf{a} + 2\mathbf{b}$ ft or $\overrightarrow{CD} = 3\mathbf{a} + 3\mathbf{b}$ ft or $\overrightarrow{OD} = 5\mathbf{a} + 5\mathbf{b}$